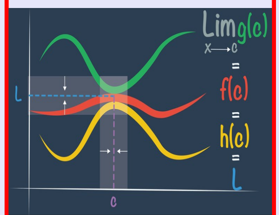
Calculus I

Lecture 21



Feb 19-8:47 AM

Class QZ 16 **Open Notes**

Verify all Conditions of the $\boxed{\text{Mean-Value theorem}}$ for $\boxed{f(x) = 2x^2 - 3x + 1}$ on $\boxed{[0,2]}$, then find all numbers c that Satisfy the Conclusion of the Mean-Value theorem.

1) $f(x)$ is a polynomial function

a) $f(x)$ is cont. $(-\infty, \infty)$

b) $f(x)$ is diff. $(-\infty, \infty)$

2) $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$f'(x) = 4x - 3$

$f'(c) = 4c - 3$

$f(2) = 2(2)^2 - 3(2) + 1 = 3$

$f(0) = 1$

$4c - 3 = \dfrac{3 - 1}{2 - 0} = \dfrac{2}{2}$

$4c - 3 = 1$

$4c = 4$

$\boxed{c = 1}$ 1 is in $(0,2)$ by Conclusion of MVT.

Jul 21-7:21 AM

Given $f(x) = 3x^4 - 16x^3 + 18x^2$, $\boxed{[-1,4]}$

1) Find $f(-1)$ & $f(4)$

$f(-1) = 37$, $f(4) = 32$

2) Find $f'(x)$, then find all critical numbers in $(-1,4)$.

$f'(x) = 12x^3 - 48x^2 + 36x$

$f'(x) = 12x(x^2 - 4x + 3)$

$f'(x) = 12x(x-3)(x-1)$

C.N. $\rightarrow f'(x) = 0$ or undefined

$f'(x) = 0 \rightarrow \boxed{x=0,\ x=3,\ x=1}$

3) Find all critical points on $(-1,4)$.

$(0,0)$, $(3,-27)$, $(1,5)$

4) Abs. Max at $x = -1 \Rightarrow (-1, 37)$ Abs. Max

Abs. Min at $x = 3 \Rightarrow (3, -27)$ Abs. Min

Jul 22-8:18 AM

$f(x) = \sqrt[3]{x^2(6-x)}$

$f'(x) = \dfrac{4-x}{\sqrt[3]{x(6-x)^2}}$, $f''(x) = \dfrac{-8}{\sqrt[3]{x^4(6-x)^5}}$

$f(4) = \sqrt[3]{16 \cdot 2} = \sqrt[3]{32} = \sqrt[3]{8}\sqrt[3]{4} = 2\sqrt[3]{4}$

1) Domain $(-\infty, \infty)$

2) VA None

3) Y-Int $(0,0)$

4) X-Ints $(0,0)$, $(6,0)$

5) Find all C.P.

$f'(x) = 0 \rightarrow 4 - x = 0 \rightarrow x = 4$

$(4, 2\sqrt[3]{4})$

$f'(x)$ is undef. at $x=0$ or $x=6$

$(0,0)$, $(6,0)$

6) Find all P.I.P.

$f''(x) = 0$ No Soln.

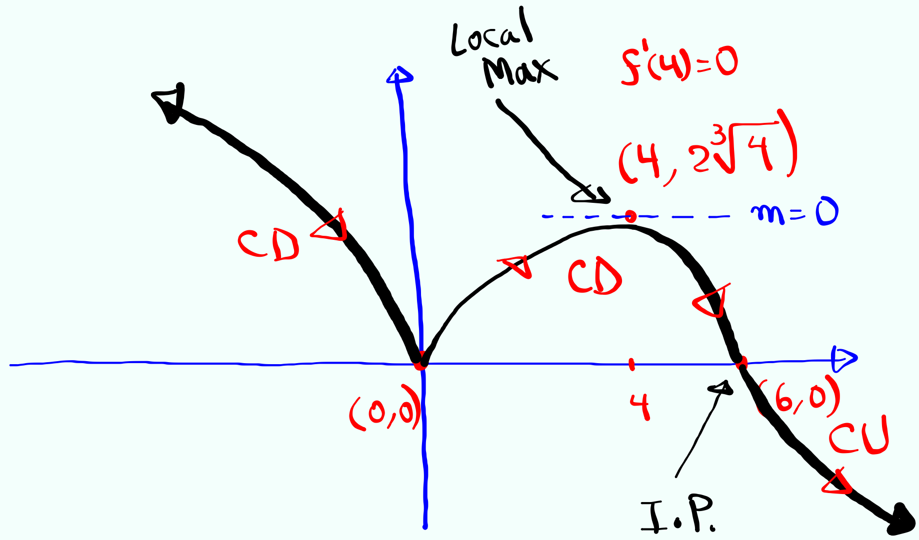
$f''(x)$ is und. at $x=0$ or $x=6$

$(0,0)$, $(6,0)$

7)

x	$-\infty$	0		4		6	∞
$f'(x)$	$-$	0	$+$	0	$-$	0	$-$
$f''(x)$	$-$	0	$-$		$-$	0	$+$
$f(x)$	↘		↗		↘	I.P.	↘

Jul 22-8:30 AM



Jul 22-8:54 AM

$$f(x)=\frac{\sqrt{2x^2+1}}{3x-5}$$

1) Domain $(-\infty,\frac{5}{3})\cup(\frac{5}{3},\infty)$

$3x-5\neq 0$

$x\neq 5/3$

2) V.A. $x=\frac{5}{3}$

3) Y-Int. $(0,\frac{-1}{5})$

4) X-Ints None

$\sqrt{2x^2+1}\neq 0$

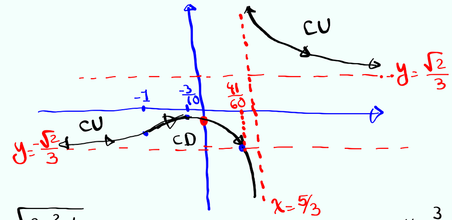
5) H.A.

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{\sqrt{2x^2+1}}{3x-5}=\frac{\infty}{\infty}$$

$$=\lim_{x\to\infty}\frac{\sqrt{2+\frac{1}{x^2}}}{3-\frac{5}{x}}=\frac{\sqrt{2}}{3}$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{-\sqrt{2+\frac{1}{x^2}}}{3-5/x}=-\frac{\sqrt{2}}{3}$$

Jul 22-8:58 AM



$f(x)=\frac{\sqrt{2x^2+1}}{3x-5}$, $f'(x)=\frac{-10x-3}{(3x-5)^2\sqrt{2x^2+1}}$ $f''(x)=\frac{2(60x^3+27x^2+34)}{(3x-5)^3\sqrt{(2x^2+1)^3}}$

1) find all C.N.

$f'(x)=0 \rightarrow -10x-3=0 \quad x=\frac{-3}{10}$

$f'(x)$ und. $\rightarrow x=\frac{5}{3}$ Not in the domain

2) find all P.I.P.

$f''(x)=0$

$60x^3+27x^2+34=0$

only one real Soln.

$x\approx -1$

$f''(x)$ und. $\rightarrow x=5/3$

Not in domain

$y=-\frac{\sqrt{2}}{3}$

$y=\frac{\sqrt{2x^2+1}}{3x-5}$ $\Rightarrow \frac{\sqrt{2x^2+1}}{3x-5}=\frac{-\sqrt{2}}{3}$

Square both Sides

$\frac{2x^2+1}{(3x-5)^2}=\frac{2}{9}$

$2(3x-5)^2=9(2x^2+1)$

$2(9x^2-30x+25)=9(2x^2+1)$

$\cancel{18x^2}-60x+50=\cancel{18x^2}+9$

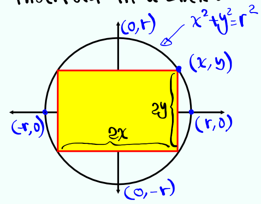
$-60x=-41$

$x=\frac{41}{60}$

$\boxed{x\approx .683}$

Jul 22-9:08 AM

Find dimensions of the rectangle with largest area that can be inscribed in a Circle of radius r.



$\text{Area}=2x\cdot 2y$

$=4xy$

$=4x\sqrt{r^2-x^2}$

$f(x)=4x\sqrt{r^2-x^2}$

$=4\sqrt{x^2(r^2-x^2)}$

$=4(r^2x^2-x^4)^{1/2}$

to optimize, we need $f'(x)$ & $f''(x)$

$f'(x)=4\cdot\frac{1}{2}(r^2x^2-x^4)^{-1/2}\cdot(2r^2x-4x^3)$

$f'(x)=\frac{2(2r^2x-4x^3)}{\sqrt{r^2x^2-x^4}}=\frac{4x(r^2-2x^2)}{\sqrt{r^2x^2-x^4}}$

$f'(x)=0 \rightarrow x=0$, $r^2-2x^2=0$

$\cancel{x=0}$ $\quad x^2=\frac{r^2}{2} \rightarrow x=\frac{r}{\sqrt{2}}$

Max $x=\frac{r}{\sqrt{2}}$

$x^2+y^2=r^2$

$\frac{r^2}{2}+y^2=r^2$

$y^2=\frac{r^2}{2}$ $y=\frac{r}{\sqrt{2}}$

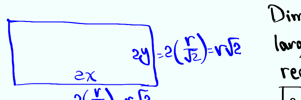
$f'>0$ $f'<0$

$x=0$ $\frac{r}{\sqrt{2}}$ $x=r$

Max.

$\text{Area}=r\sqrt{2}\cdot r\sqrt{2}$

$\boxed{2r^2}$



Dimensions of largest area rectangle are $\boxed{r\sqrt{2}}$ by $\boxed{r\sqrt{2}}$

Jul 22-9:27 AM

Integration:

1) $\int c\, f(x)\,dx = c\int f(x)\,dx$

2) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

3) $\int K\,dx = Kx + C$

4) $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C,\ n \neq -1$

5) $\int \sin x\,dx = -\cos x + C$

6) $\int \cos x\,dx = \sin x + C$

7) $\int \sec^2 x\,dx = \tan x + C$

8) $\int \csc^2 x\,dx = -\cot x + C$

9) $\int \sec x \tan x\,dx = \sec x + C$

10) $\int \csc x \cot x\,dx = -\csc x + C$

ex: $\int [10x^3 - 4\sec^2 x]\,dx = \int 10x^3\,dx - \int 4\sec^2 x\,dx$

$= 10\int x^3\,dx - 4\int \sec^2 x\,dx = 10\cdot\dfrac{x^4}{4} - 4\cdot\tan x + C$

$= \boxed{\frac{5}{2}x^4 - 4\tan x + C}$

ex: $\int (6x - 5\sin x + 8)\,dx$

$= 6\cdot\dfrac{x^2}{2} - 5\cdot(-\cos x) + 8x + C = \boxed{3x^2 + 5\cos x + 8x + C}$

Jul 22-10:06 AM

Find $\int (x+4)(2x-3)\,dx$

$= \int [2x^2 + 5x - 12]\,dx$

$= 2\cdot\dfrac{x^3}{3} + 5\cdot\dfrac{x^2}{2} - 12x + C$

$= \boxed{\frac{2}{3}x^3 + \frac{5}{2}x^2 - 12x + C}$

Find $\int \dfrac{x^3 - 2\sqrt{x}}{x}\,dx = \int \left[\dfrac{x^3}{x} - \dfrac{2\sqrt{x}}{x}\right]dx$

$= \int \left[x^2 - \dfrac{2}{\sqrt{x}}\right]dx = \int [x^2 - 2x^{-1/2}]\,dx$

$= \dfrac{x^3}{3} - 2\cdot\dfrac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$

$= \frac{1}{3}x^3 - 2\cdot\dfrac{x^{1/2}}{1/2} + C$

$= \frac{1}{3}x^3 - 2\cdot 2\sqrt{x} + C = \boxed{\frac{1}{3}x^3 - 4\sqrt{x} + C}$

Find $\int (\underbrace{1 + \tan^2 x})\,dx = \int \sec^2 x\,dx = \boxed{\tan x + C}$

Jul 22-10:18 AM

Defenite integral

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F'(x) = f(x)$

$$\int_1^2 x^3\,dx = \frac{x^4}{4}\Big|_1^2 = \frac{2^4}{4} - \frac{1^4}{4} = \frac{16}{4} - \frac{1}{4} = \boxed{\frac{15}{4}}$$

Evaluate

$$\int_0^3 (3x^2+1)\,dx = \left[3\frac{x^3}{3} + 1x\right]\Big|_0^3$$

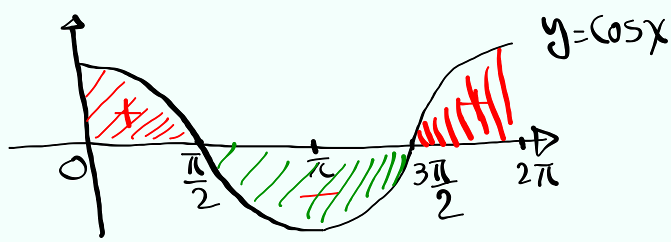
$$= (x^3+x)\Big|_0^3 = [3^3+3] - [0^3+0]$$

$$= \boxed{30}$$

Jul 22-10:26 AM

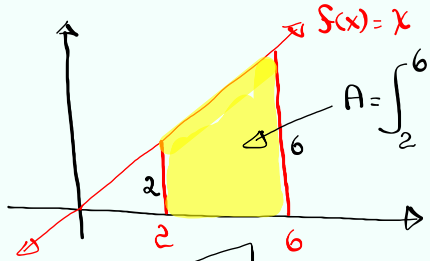
Evaluate $\displaystyle\int_0^{2\pi} \cos x\,dx = \sin x\Big|_0^{2\pi}$

$$= \cancel{\sin 2\pi}^{\,0} - \cancel{\sin 0}^{\,0} = \boxed{0}$$



Jul 22-10:33 AM

Find the area below $\boxed{f(x)=x}$ ($y=x$), above x-axis from $x=2$ to $x=6$

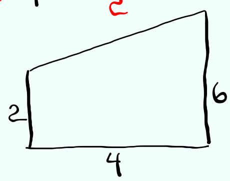


$$A=\int_2^6 x\,dx=\frac{x^2}{2}\Big|_2^6$$

$$=\frac{1}{2}\left[6^2-2^2\right]$$

$$=\frac{1}{2}\left[36-4\right]$$

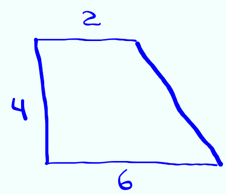
$$=\frac{1}{2}\cdot 32=\boxed{16}$$



Trapezoid

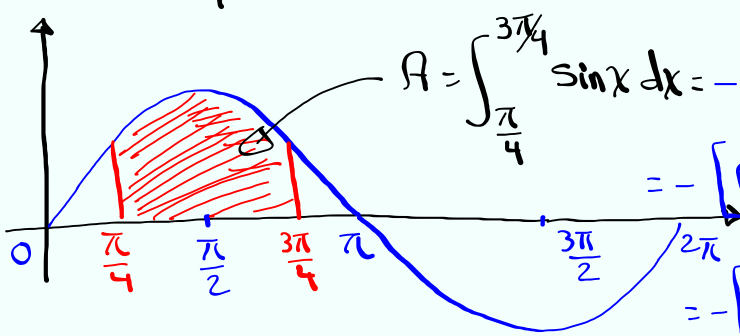
$$A=\frac{h(B+b)}{2}$$

$$=\frac{4(6+2)}{2}=\frac{4\cdot 8}{2}=\frac{32}{2}=16$$



Jul 22-10:39 AM

Find the area below $f(x)=\sin x$, above x-axis, from $x=\frac{\pi}{4}$ to $x=\frac{3\pi}{4}$. Drawing Required.



$$A=\int_{\frac{\pi}{4}}^{3\pi/4}\sin x\,dx=-\cos x\Big|_{\pi/4}^{3\pi/4}$$

$$=-\left[\cos\frac{3\pi}{4}-\cos\frac{\pi}{4}\right]$$

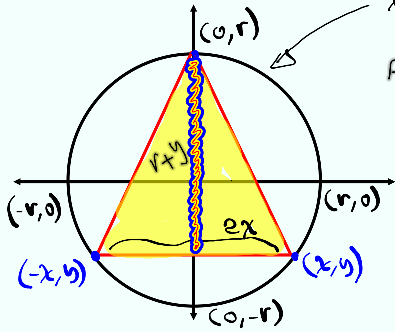
$$=-\left[-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\right]$$

$$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}=\frac{2\sqrt{2}}{2}$$

$$=\boxed{\sqrt{2}}$$

Jul 22-10:45 AM

Find the dimensions of the isosceles triangle of the largest area that can be inscribed in a circle of radius r.



$x^2+y^2=r^2$

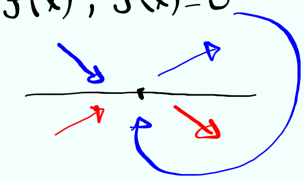
$A = \frac{bh}{2} = \frac{\cancel{2}x(r+y)}{\cancel{2}}$

$A = rx + xy$

$A = rx + x\sqrt{r^2-x^2}$

$f(x) = rx + x\sqrt{r^2-x^2}$

$f'(x)$, $f'(x)=0$



Ans. is.
Base $r\sqrt{3}$
Height $\frac{3r}{2}$

?

Jul 22-10:52 AM